Approximate unconventional geometric phase gate by highly squeezed operators with a cavity QED system

2006 J. Phys. A: Math. Gen. 39 2435

(http://iopscience.iop.org/0305-4470/39/10/012)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **39** (2006) 2435–2442 doi:10.1088/0305-4470/39/10/012

Approximate unconventional geometric phase gate by highly squeezed operators with a cavity QED system

Chang-Yong Chen[1,2,3] **and Ke-Lin Gao**[2,3]

[1] Department of Physics and Information Engineering, Hunan Institute of Humanities, Science and Technology, Loudi 417000, People's Republic of China
[2] State Key Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, Wuhan 430071, People's Republic of China
[3] Graduate School of the Chinese Academy of Science, Beijing 100049, People's Republic of China

E-mail: chenchangyong_64@hotmail.com

Received 28 November 2005, in final form 27 January 2006
Published 22 February 2006
Online at stacks.iop.org/JPhysA/39/2435

Abstract
We introduce the general definition of the unconventional geometric phase independent of the specific physical system and show how a highly squeezed operator can approximately induce the general unconventional geometric phase, which goes beyond the original unconventional one by displacement operators (Zhu and Wang 2003 *Phys. Rev. Lett.* **91** 187902). By means of the squeezed operator concerning the cavity mode state along a closed path in the phase space, we discuss specifically how to implement approximately a two-qubit geometric phase gate in a cavity QED system with two-photon interaction between the atoms and the cavity mode, assisted by a classical field. Discussions regarding the implementation time of the gating, the possible decaying sources and the experimental feasibility are given in detail.

PACS numbers: 03.67.Lx, 03.65.Ta, 42.50.Dv

1. Introduction

It is well known that the construction of a large-scale quantum computer requires the suppression of decoherence from variable parameter fluctuations. Since quantum gates based on the dynamical phases are very sensitive to the parameter fluctuations in the operations, the ideas of decoherence-free subspace [1] and the geometric phase [2–9] have been considered to be promising for achieving built-in fault-tolerant quantum gates.

Our investigation in this paper is related to geometric quantum gatings (GQGs), which can be classified to be of two kinds. The conventional geometric quantum gates [2–8], determined by the global geometric features, offer potential advantages in resistance to parameter fluctuations. But generally they require some additional operations for avoiding the effects from the corresponding dynamical phases, which may bring about additional errors. Alternatively, we may choose the unconventional GQG [9, 10], which not only possesses all geometric advantages owned by the conventional geometric gate but also is independent of the initial state of the system. So implementation of unconventional GQGs should reach high fidelity. Although the unconventional GQG [9] also creates the geometric phase with a nonzero dynamic one, it is not necessary to eliminate the dynamic phase because the dynamic phase is proportional to the geometric phase by a constant independent of the parameters of the qubit system [9]. We have noted some quantum computing schemes based on the ideas of conventional GQGs by using super-conducting nanocircuits [4], NMR [6], semiconductor nanostructure [7] and trapped ions [8]. There has been actually no experiment achieved for the conventional GQG. In contrast, for the unconventional GQG, besides the theoretical proposals [9, 10], an experiment has been done with trapped ions [11]. The unconventional GQGs proposed so far are based on the displacement operator $\widehat{D}(\alpha)$ in the phase space. A natural question arises: whether a highly squeezed operator $\widehat{S}(\varepsilon)$ can do the same job.

In this paper, we first introduce the general definition of the unconventional geometric phase, independent of specific physical systems, and then show how a highly squeezed operator induces approximately the general unconventional GQG, which is different from the original unconventional one defined in [9]. A two-qubit unconventional GQG will be specifically investigated in a cavity QED system by means of two-photon interaction between the atoms and the cavity mode, assisted by a classical field. We find that our gating is related to the population of the cavity mode, and is thereby sensitive to the cavity decay. Nevertheless, with the cavity initially prepared in a vacuum state, our scheme is robust to both the cavity decay and the atomic spontaneous emission.

This paper is organized as follows. In section 2, the definition of the general unconventional geometric phase caused by the squeezed operator along a closed path in the phase space is proposed. In section 3, the dynamical evolution of two identical three-level atoms is studied in detail in the interaction with a quantized cavity mode and a classical field under the condition of multi-photon interaction. In section 4, the two-qubit unconventional geometric phase gate is carried out in the cavity system. Finally, some discussions and conclusion are given in section 5.

2. General unconventional geometric phase by the squeezed operator

The definition of the general geometric phase shift due to displacement along an arbitrary path in the squeezing parameter phase space is given in this section, as a generalization of the definition of the geometric phase shift in [12, 13]. The lth-order squeezed operator can be expressed as follows:

$$\widehat{S}(\varepsilon) = \mathrm{e}^{\frac{1}{2}\varepsilon^*\widehat{a}^l - \frac{1}{2}\varepsilon\widehat{a}^{+l}}, \tag{1}$$

where $\widehat{a}$ and $\widehat{a}^+$ are the annihilation and creation operators of the harmonic oscillator (for example, cavity modes in the case of the cavity QED system), respectively, and $\varepsilon = r\,\mathrm{e}^{-2\mathrm{i}\theta}$ is the squeezing parameter and may displace in the squeezing parameter phase space. If $|\mathrm{d}\varepsilon_1|$ and $|\mathrm{d}\varepsilon_2|$ are approaching zero, the squeezed operators are approximately satisfied with

$$\widehat{S}(\mathrm{d}\varepsilon_1)\widehat{S}(\mathrm{d}\varepsilon_2) \approx \widehat{S}(\mathrm{d}\varepsilon_1 + \mathrm{d}\varepsilon_2)\,\mathrm{e}^{\frac{\mathrm{i}}{4}\mathrm{Im}(\mathrm{d}\varepsilon_1^*\mathrm{d}\varepsilon_2)[\widehat{a}^l,\widehat{a}^{+l}]}, \tag{2}$$

where the higher order small terms are neglected. For a path consisting of N short straight sections $\mathrm{d}\varepsilon_j$, the total operator is

$$\widehat{S}_t = \widehat{S}(\mathrm{d}\varepsilon_N)\cdots\widehat{S}(\mathrm{d}\varepsilon_1) \approx \widehat{S}\left(\sum_{j=1}^{N}\mathrm{d}\varepsilon_j\right)\exp\left\{\frac{\mathrm{i}}{4}\mathrm{Im}\left(\sum_{j=2}^{N}\mathrm{d}\varepsilon_j\sum_{k=1}^{j-1}\mathrm{d}\varepsilon_k^*\right)[\widehat{a}^{l},\widehat{a}^{+l}]\right\}. \quad (3)$$

An arbitrary path γ can be reached in the limit $N \to \infty$. We have

$$\widehat{S}_t \approx \widehat{S}\left(\int_\gamma \mathrm{d}\varepsilon\right)\mathrm{e}^{\mathrm{i}\widehat{\Theta}_l}, \quad (4)$$

where

$$\widehat{\Theta}_l \approx \frac{1}{4}\,\mathrm{Im}\left(\int_\gamma \varepsilon^*\mathrm{d}\varepsilon\right)[\widehat{a}^{l},\widehat{a}^{+l}]. \quad (5)$$

For a closed path and an initial Fock state, we have

$$\widehat{S}_t \approx \widehat{S}(0)\,\mathrm{e}^{\mathrm{i}\widehat{\Theta}_l} = \mathrm{e}^{\mathrm{i}\widehat{\Theta}_l}, \quad (6)$$

$$\widehat{\Theta}_l \approx \frac{1}{4}\mathrm{Im}\left(\oint \varepsilon^*\mathrm{d}\varepsilon\right)[\widehat{a}^{l},\widehat{a}^{+l}], \quad (7)$$

$$\Theta_l = \langle n|\widehat{\Theta}_l|n\rangle \approx \frac{1}{4}\left(\frac{(n+l)!}{n!} - \frac{n!}{(n-l)!}\right)\mathrm{Im}\left(\oint \varepsilon^*\mathrm{d}\varepsilon\right), \quad (8)$$

where the phase operator Θ_l is determined by both the area involved by the loop in the squeezing parameter phase space and the quantized state of the harmonic oscillator. This is different from that of [12, 13], in which the geometric phase produced by the displacement operator is independent of the quantized state of the harmonic oscillator. When $l = 1$ and $\varepsilon = -2\alpha$ are satisfied, our definition reduces to that of [12, 13]:

$$\widehat{D}_t = \widehat{D}(0)\,\mathrm{e}^{\mathrm{i}\widehat{\Theta}_1} = \mathrm{e}^{\mathrm{i}\widehat{\Theta}_1}, \quad \Theta_1 = \langle n|\,\widehat{\Theta}_1\,|n\rangle = \mathrm{Im}\left(\oint \alpha^*\mathrm{d}\alpha\right). \quad (9)$$

In this paper, for a specific study, we will focus on the case of $l = 2$, where

$$\widehat{S}_t \approx \widehat{S}(0)\,\mathrm{e}^{\mathrm{i}\widehat{\Theta}_2} = \mathrm{e}^{\mathrm{i}\widehat{\Theta}_2}, \quad \Theta_2 = \langle n|\,\widehat{\Theta}_2\,|n\rangle \approx \left(n+\frac{1}{2}\right)\mathrm{Im}\left(\oint \varepsilon^*\mathrm{d}\varepsilon\right), \quad (10)$$

where n is the eigenvalue of the population operator of the harmonic oscillator. We mention that the definition of the general unconventional geometric phase is independent of any specific physical systems. The above equations imply that any highly squeezed operators can approximately induce the unconventional geometric phase shifts.

3. Dynamics of the cavity QED system with multi-photon interaction

We study two identical three-level atoms, each of which has one excited state $|i\rangle$ and two ground states $|e\rangle$ and $|g\rangle$. The qubits are encoded in the states $|e\rangle$ and $|g\rangle$, and the state $|i\rangle$ is an auxiliary state. The transition $|e\rangle \to |i\rangle$ is an l-photon process, driven by the cavity mode with coupling constant g and detuning $\Delta = \omega_0 - \omega_c$, assisted by a classical laser field with Rabi frequency Ω and detuning $\Delta - \delta = \omega_0 - \omega_L$, with $\delta \ll \Delta$. In fact, $\delta = \omega_L - \omega_c$. As $|g\rangle$ is not involved in the interaction, the Hamiltonian [10] can be expressed as (assuming $\hbar = 1$)

$$\widehat{H}_{sl} = \omega_0\sum_{j=1,2}\widehat{S}_{z,j} + \omega_c\widehat{a}^{+}\widehat{a} + g\sum_{j=1,2}\left(\widehat{a}^{+l}\widehat{S}_j^{-} + \widehat{a}^{l}\widehat{S}_j^{+}\right) + \Omega\sum_{j=1,2}\left(\mathrm{e}^{-\mathrm{i}\omega_L t}\widehat{S}_j^{+} + \mathrm{e}^{\mathrm{i}\omega_L t}\widehat{S}_j^{-}\right), \quad (11)$$

where ω_0, ω_c and ω_L are the frequencies of the resonant transition between $|e\rangle$ and $|i\rangle$, the cavity mode and the classical laser field, respectively. $\widehat{S}_{z,j} = \frac{1}{2}(|i\rangle\langle i| - |e\rangle\langle e|)$, $\widehat{S}_j^+ = |i\rangle\langle e|$ and $\widehat{S}_j^- = |e\rangle\langle i|$, $\widehat{a}^+$ and $\widehat{a}$ are the creation and annihilation operators for the cavity mode, respectively, and l is an integer. In the rotating frame with respect to the cavity frequency ω_c, the Hamiltonian is given by

$$\widehat{H}_{il} = \sum_{j=1,2}\left[\Delta\widehat{S}_{z,j} + (g\widehat{a}^{+l}\,\mathrm{e}^{\mathrm{i}(l-1)\omega_c t} + \Omega\,\mathrm{e}^{\mathrm{i}\delta t})\widehat{S}_j^- + (g\widehat{a}^{l}\,\mathrm{e}^{-\mathrm{i}(l-1)\omega_c t} + \Omega\,\mathrm{e}^{-\mathrm{i}\delta t})\widehat{S}_j^+\right]. \tag{12}$$

In the case that $\Delta \gg \Omega, g$, without exchange of energy between atoms and the fields, the Hamiltonian of equation (11) can be replaced by an effective Hamiltonian

$$\begin{aligned}\widehat{H}_{iel} = \sum_{j=1,2}\frac{1}{\Delta}&\left\{\left[\left(\frac{(n+l)!}{n!} - \frac{n!}{(n-l)!}\right)g^2\widehat{a}^{+l}\widehat{a}^{l} + \Omega^2 + \Omega g\widehat{a}^{l}\,\mathrm{e}^{\mathrm{i}\delta' t} + \Omega g\widehat{a}^{+l}\,\mathrm{e}^{-\mathrm{i}\delta' t}\right]\right.\\ &\left.\times(|i_j\rangle\langle i_j| - |e_j\rangle\langle e_j|)] + \left[\frac{(n+l)!}{n!} - \frac{n!}{(n-l)!}\right]g^2|i_j\rangle\langle i_j|[\widehat{a}^{l},\widehat{a}^{+l}]\right\}\\ &+\frac{1}{\Delta}\left[\frac{(n+l)!}{n!} - \frac{n!}{(n-l)!}\right]g^2\left(\widehat{S}_1^+\widehat{S}_2^- + \widehat{S}_1^-\widehat{S}_2^+\right)[\widehat{a}^{l},\widehat{a}^{+l}],\end{aligned} \tag{13}$$

where n is the population of the cavity mode and $\delta' = \omega_L - l\omega_c$. In this paper, we focus on the case that two-photon interaction occurs, i.e., $l = 2$. Therefore, $\delta' = \omega_L - 2\omega_c$. Then the corresponding effective Hamiltonian may be expressed as

$$\begin{aligned}\widehat{H}_{ie2} = \sum_{j=1,2}\frac{1}{\Delta}&\{[2(2n+1)g^2\widehat{a}^{+2}\widehat{a}^{2} + \Omega^2 + \Omega g\widehat{a}^{2}\,\mathrm{e}^{\mathrm{i}\delta' t} + \Omega g\widehat{a}^{+2}\,\mathrm{e}^{-\mathrm{i}\delta' t}](|i_j\rangle\langle i_j| - |e_j\rangle\langle e_j|)\\ &+2(2n+1)g^2|i_j\rangle\langle i_j|[\widehat{a}^{2},\widehat{a}^{+2}]\} + \frac{2(2n+1)g^2}{\Delta}\left(\widehat{S}_1^+\widehat{S}_2^- + \widehat{S}_1^-\widehat{S}_2^+\right)[\widehat{a}^{2},\widehat{a}^{+2}].\end{aligned} \tag{14}$$

We assume that $\widehat{H}_{ie2} = \widehat{H}_{02} + \widehat{H}'_{ie2}$, where

$$\widehat{H}_{02} = \sum_{j=1,2}\frac{1}{\Delta}\{[2(2n+1)g^2\widehat{a}^{+2}\widehat{a}^{2} + \Omega^2](|i_j\rangle\langle i_j| - |e_j\rangle\langle e_j|) + [2(2n+1)g^2|i_j\rangle\langle i_j|[\widehat{a}^{2},\widehat{a}^{+2}]\}, \tag{15}$$

$$\begin{aligned}\widehat{H}'_{ie2} = \sum_{j=1,2}\frac{1}{\Delta}&\left\{[\Omega g\widehat{a}^{2}\,\mathrm{e}^{\mathrm{i}\delta' t} + \Omega g\widehat{a}^{+2}\,\mathrm{e}^{-\mathrm{i}\delta' t}](|i_j\rangle\langle i_j| - |e_j\rangle\langle e_j|)\right.\\ &\left.+\frac{2(2n+1)g^2}{\Delta}\left(\widehat{S}_1^+\widehat{S}_2^- + \widehat{S}_1^-\widehat{S}_2^+\right)[\widehat{a}^{2},\widehat{a}^{+2}]\right\}.\end{aligned} \tag{16}$$

Performing the unitary transformation $|\psi(t)\rangle = \mathrm{e}^{-\mathrm{i}H_{02}t}|\psi'(t)\rangle$, from the Schrődinger equations $\mathrm{i}\,\mathrm{d}|\psi(t)\rangle/\mathrm{d}t = H_{ie2}|\psi(t)\rangle$ and $\mathrm{i}\,\mathrm{d}|\psi'(t)\rangle/\mathrm{d}t = H'_{i2}|\psi'(t)\rangle$, we obtain

$$\begin{aligned}\widehat{H}'_{i2} &= \mathrm{e}^{\mathrm{i}\widehat{H_{02}}t}\widehat{H}'_{ie2}\,\mathrm{e}^{-\mathrm{i}\widehat{H}_{02}t}\\ &= \sum_{j=1,2}\frac{\Omega g}{\Delta}\left\{\left[\widehat{a}^{2}\exp\left(\mathrm{i}\left[\delta' - \frac{8(2n+1)(n+1)g^2}{\Delta}\right]t\right)\right.\right.\\ &\quad\left.+\,\widehat{a}^{+2}\exp\left(-\mathrm{i}\left[\delta' - \frac{8(2n+1)(n+1)g^2}{\Delta}\right]t\right)\right]|i_j\rangle\langle i_j|\end{aligned}$$

$$- \left[\widehat{a}^2 \exp\left(\mathrm{i}\left[\delta' + \frac{4(2n+1)^2g^2}{\Delta}\right]t\right) + \widehat{a}^{+2} \exp\left(-\mathrm{i}\left[\delta' + \frac{4(2n+1)^2g^2}{\Delta}\right]t\right)\right] |e_j\rangle\langle e_j|\} + \frac{2(2n+1)g^2}{\Delta}\left(\widehat{S}_1^+\widehat{S}_2^- + \widehat{S}_1^-\widehat{S}_2^+\right)[\widehat{a}^2, \widehat{a}^{+2}], \tag{17}$$

in which the only term that contributes an unconventional geometric phase shift to the evolution of the qubit states $|g_j\rangle$ and $|e_j\rangle$ is

$$\widehat{H}'_{ig2} = -\frac{\Omega g}{\Delta}\left[\widehat{a}^2 \exp\left(\mathrm{i}\left[\delta' + \frac{4(2n+1)^2g^2}{\Delta}\right]t\right) + \widehat{a}^{+2} \exp\left(-\mathrm{i}\left[\delta' + \frac{4(2n+1)^2g^2}{\Delta}\right]t\right)\right] \sum_{j=1,2} |e_j\rangle\langle e_j|. \tag{18}$$

According to the definition of the squeezed operator, during the infinitesimal interval $[t, t + \mathrm{d}t]$, the corresponding evolution of the states of our encoded space will be decided by

$$\widehat{S}(\mathrm{d}\varepsilon) = \mathrm{e}^{-\mathrm{i}\widehat{H}'_{ig2}\mathrm{d}t}, \tag{19}$$

where

$$\mathrm{d}\varepsilon = -\mathrm{i}\frac{2\Omega g}{\Delta} \exp\left(-\mathrm{i}\left[\delta' + \frac{4(2n+1)^2g^2}{\Delta}\right]t\right)\mathrm{d}t. \tag{20}$$

It is easy to see that the qubit states including a single state $|e_j\rangle$ will evolve according to $\widehat{S}(\mathrm{d}\varepsilon)$, and the one with two $|e_j\rangle$ will evolve according to $\widehat{S}(2\mathrm{d}\varepsilon)$. Other states will remain unchanged. Straightforwardly, after an interaction time t', the squeezed parameter ε can be expressed as

$$\begin{aligned}\varepsilon &= -\mathrm{i}\frac{2\Omega g}{\Delta}\int_0^{t'} \exp\left(-\mathrm{i}\left[\delta' + \frac{4(2n+1)^2g^2}{\Delta}\right]t\right)\mathrm{d}t \\ &= \frac{2\Omega g}{\Delta\delta' + 4(2n+1)^2g^2}\left[\exp\left(-\mathrm{i}\left[\delta' + \frac{4(2n+1)^2g^2}{\Delta}\right]t'\right) - 1\right],\end{aligned} \tag{21}$$

and the geometric phase shifts $\Theta_2 = \langle n|\widehat{\Theta}_2|n\rangle$ and $\Theta'_2 = \langle n|\widehat{\Theta}'_2|n\rangle$, regarding $|e_j\rangle|g_k\rangle(|g_j\rangle|e_k\rangle)$ and $|e_j\rangle|e_k\rangle(j \neq k)$, respectively, are

$$\begin{aligned}\Theta_2 = \langle n|\widehat{\Theta}_2|n\rangle &= \left(n + \frac{1}{2}\right)\mathrm{Im}\left(\int_\gamma \varepsilon'^*\mathrm{d}\varepsilon'\right) = \frac{(2n+1)(\Omega g)^2}{2\Delta[\Delta\delta' + 4(2n+1)^2g^2]} \\ &\times \left[t' - \frac{1}{\delta' + \frac{4(2n+1)^2g^2}{\Delta}} \sin\left(\delta' + \frac{4(2n+1)^2g^2}{\Delta}\right)t'\right],\end{aligned} \tag{22}$$

and

$$\Theta'_2 = \left(n + \frac{1}{2}\right)\mathrm{Im}\left(\int_\gamma 2\varepsilon'^*\mathrm{d}2\varepsilon'\right) = 4\Theta_2. \tag{23}$$

4. Unconventional geometric phase gates in two-photon interaction

We consider that the cavity is initially in the Fock state $|0\rangle$ or$|1\rangle$, and we pay our attention on the second-order squeezing process. When the squeezed parameter $\mathrm{d}\varepsilon_2$ moves along a closed path, returning to the original point in the phase space, a global geometric phase conditional upon the electronic states and the cavity mode states will appear. From equation (21), the condition of the system along the closed path is

$$\left|\delta' + \frac{4(2n+1)^2 g^2}{\Delta}\right| t' = 2k\pi, \tag{24}$$

where k is an integer, determined by specific parameters in the system. Therefore, we can give the evolution expression of the system, governed by the Hamiltonian $\widehat{H}_{02}$,

$$\begin{aligned}
&|g_1\rangle|g_2\rangle|n\rangle \rightarrow |g_1\rangle|g_2\rangle|n\rangle \rightarrow |g_1\rangle|g_2\rangle|n\rangle,\\
&|g_1\rangle|e_2\rangle|n\rangle \rightarrow \mathrm{e}^{\mathrm{i}\Omega^2 t'/\Delta}\,\mathrm{e}^{\mathrm{i}\Theta_2} S(\varepsilon_2)|g_1\rangle|e_2\rangle|n\rangle \rightarrow \mathrm{e}^{\mathrm{i}(\Theta_2+\Omega^2 t'/\Delta)}|g_1\rangle|e_2\rangle|n\rangle,\\
&|e_1\rangle|g_2\rangle|n\rangle \rightarrow \mathrm{e}^{\mathrm{i}\Omega^2 t'/\Delta}\,\mathrm{e}^{\mathrm{i}\Theta_2} S(\varepsilon_2)|e_1\rangle|g_2\rangle|n\rangle \rightarrow \mathrm{e}^{\mathrm{i}(\Theta_2+\Omega^2 t'/\Delta)}|e_1\rangle|g_2\rangle|n\rangle,\\
&|e_1\rangle|e_2\rangle|n\rangle \rightarrow \mathrm{e}^{\mathrm{i}2\Omega^2 t'/\Delta}\,\mathrm{e}^{\mathrm{i}\Theta_2'} S(2\varepsilon_2)|e_1\rangle|e_2\rangle|n\rangle \rightarrow \mathrm{e}^{\mathrm{i}(4\Theta_2+2\Omega^2 t'/\Delta)}|e_1\rangle|e_2\rangle|n\rangle,
\end{aligned} \tag{25}$$

where

$$\Theta_2 = \frac{(2n+1)(\Omega g)^2}{2\Delta[\Delta\delta' + 4(2n+1)^2 g^2]} t'', \qquad \Theta_2' = 4\Theta_2 \quad (n = 0, 1). \tag{26}$$

After the single qubit operation is performed $|e_j\rangle \rightarrow \exp\left(-\mathrm{i}\left(\Theta_2 + \frac{\Omega^2}{\Delta} t'\right)\right)|e_j\rangle$, the evolution turns to

$$\begin{aligned}
&|g_1\rangle|g_2\rangle|n\rangle \rightarrow |g_1\rangle|g_2\rangle|n\rangle, \qquad |g_1\rangle|e_2\rangle|n\rangle \rightarrow |g_1\rangle|e_2\rangle|n\rangle,\\
&|e_1\rangle|g_2\rangle|n\rangle \rightarrow |e_1\rangle|g_2\rangle|n\rangle, \qquad |e_1\rangle|e_2\rangle|n\rangle \rightarrow \mathrm{e}^{\mathrm{i}2\Theta_2}|e_1\rangle|e_2\rangle|n\rangle.
\end{aligned} \tag{27}$$

It is clear that this is an unconventional geometric $2\Theta_2$-phase gate based on the squeezed operator and the cavity QED system. If $|2\Theta_2| = \pi$ is satisfied, we can acquire a π-phase gate. Therefore, this unconventional geometric phase gate can be implemented in the cavity QED system from an initial Fock state $|n\rangle$.

5. Discussion and conclusion

Although the excited state $|i\rangle$ is not actually populated throughout the scheme, which suppresses the spontaneous emission, the phase shift including the population of the cavity mode restricts our scheme to be applied in general situation. To avoid the cavity decay to the best, we have to prepare the cavity to be initially in a vacuum state. Under the condition $(\Omega g)/(\Delta\delta' + 4g^2) \ll 1$, i.e., $S(\varepsilon) \backsimeq 1$, the detrimental effect from the cavity decay on the gating is negligible. If we assume $\omega_0 = 25g$, $\omega_L = 16g$, $\omega_c = 15g$ and $\Omega = g$, then we have $\Delta = 10g$ and $\delta' = -14g$. In the case of $n = 0$, the required time to implement our gate is $t'' = \Delta|\Delta\delta' + 4g^2|\pi/(\Omega g)^2 = 1360\pi g^{-1}$. Following the idea in [10], we take the decay rate of the cavity to be $\gamma = g/27$ [14, 15]. It is easy to find that the gate error due to the cavity decay is about 3.42%.

We now turn to discuss the influence of the approximation in equation (2) on the geometric phase shift. When $|\mathrm{d}\varepsilon_i|(i = 1, 2) \rightarrow 0$, we can obtain the geometric phase shift error

$$\eta_e = \tfrac{1}{3}[\mathrm{d}\varepsilon_1\,\mathrm{d}\varepsilon_1^* + \mathrm{d}\varepsilon_2\,\mathrm{d}\varepsilon_2^* + \mathrm{d}\varepsilon_1\,\mathrm{d}\varepsilon_2^* + \mathrm{d}\varepsilon_1^*\,\mathrm{d}\varepsilon_2]. \tag{28}$$

Setting $|\mathrm{d}\varepsilon_1| = |\mathrm{d}\varepsilon_2| \leqslant 0.1(\mathrm{or}0.01)$, we have $\eta_e = 1.33\%$ (or1.33×10^{-4}). Therefore, if the squeezed parameter $\mathrm{d}\varepsilon_i\,(i = 1, \ldots, N) \leqslant 10^{-4}$ of each step in equation (3) can be controlled, with the assumption that $N = 10^3$, we estimate that its resulting additive geometric phase shift error is $\eta_{eadd} = N^2\eta_{e(\mathrm{d}\varepsilon_i)} \leqslant 1.33 \times 10^{-2}$. In fact, the $|\mathrm{d}\varepsilon| \rightarrow 0$ in equation (20) can be effectively controlled by means of adjusting parameters Δ, Ω, g and $\mathrm{d}t$. Therefore, in this case, the gating time will be correspondingly a little prolonged.

The scheme can be directly extended to the cavity field initially in a many-photon state. Although more photons are involved in the interaction, the more difficult is the experimental realization. Nevertheless, in the case that the cavity state is initially in an arbitrary Fock $|n\rangle (n \neq 0, 1)$, our scheme works by only changing the single qubit rotation $|e_j\rangle \rightarrow \exp\left(-\mathrm{i}\left(\Theta_2 + \frac{\Omega^2}{\Delta} t'\right)\right)|e_j\rangle$ to $|e_j\rangle \rightarrow \exp\left(-\mathrm{i}\left(\Theta_2 + \frac{\Omega'^2}{\Delta} t'\right)\right)|e_j\rangle$, where $\Omega'^2 = 2n(n-1)(2n+1)g^2/\Delta + \Omega^2$. Moreover, our scheme can be transcribed to an ion-trap system since the mathematics, based on the Jaynes–Cummings model, for cavity-QED and ion trap are somewhat analogous, although the physics behind the mathematics is different. In ion-trap systems, we need the detuned lasers to radiate the ions. Following the idea in [11], we can also achieve the unconventional GQG induced by the squeezed operator in an ion trap by properly choosing multi-phonon detunings.

Actually, after direct but complex algebra, the expressions of the squeezed parameter ε_l, the corresponding geometric phase shifts Θ_l and Θ_l', induced by the lth squeezed operator, can be shown as follows, respectively,

$$\begin{aligned}\varepsilon_l &= -\mathrm{i}\frac{2\Omega g}{\Delta}\int_0^{t'} \exp\left(-\mathrm{i}\left[\delta' + \frac{\left(\frac{(n+l)!}{n!} - \frac{n!}{(n-l)!}\right)^2 g^2}{\Delta}\right] t\right) \mathrm{d}t \\ &= \frac{2\Omega g}{\Delta\delta' + 4(2n+1)^2 g^2}\left[\exp\left(-\mathrm{i}\left[\delta' + \frac{\left(\frac{(n+l)!}{n!} - \frac{n!}{(n-l)!}\right)^2 g^2}{\Delta}\right] t'\right) - 1\right], \end{aligned} \tag{29}$$

$$\begin{aligned}\Theta_l &= \frac{1}{4}\left(\frac{(n+l)!}{n!} - \frac{n!}{(n-l)!}\right) \mathrm{Im}\left(\left(\int_\gamma \varepsilon_l'^*\right)\mathrm{d}\varepsilon_l'\right) \\ &= \frac{\left(\frac{(n+l)!}{n!} - \frac{n!}{(n-l)!}\right)(\Omega g)^2}{4\Delta\left[\Delta\delta + \left(\frac{(n+l)!}{n!} - \frac{n!}{(n-l)!}\right)^2 g^2\right]} \\ &\quad\times\left[t' - \frac{1}{\delta' + \frac{\left(\frac{(n+l)!}{n!} - \frac{n!}{(n-l)!}\right)^2 g^2}{\Delta}} \sin\left(\delta + \frac{\left(\frac{(n+l)!}{n!} - \frac{n!}{(n-l)!}\right)^2 g^2}{\Delta}\right) t'\right]\end{aligned} \tag{30}$$

and

$$\begin{aligned}\Theta_l' &= \frac{1}{4}\left(\frac{(n+l)!}{n!} - \frac{n!}{(n-l)!}\right) \mathrm{Im}\left[\int_\gamma (2\varepsilon_l'^*)\mathrm{d}(2\varepsilon_l')\right] \\ &= 4\Theta_l. \end{aligned} \tag{31}$$

Consider that the cavity is initially in Fock states $|0\rangle, |1\rangle, \ldots, |n-1\rangle$, respectively. Through the same process as in section 4, and by means of the single qubit operation $|e_j\rangle \rightarrow \exp\left(-\mathrm{i}\left(\Theta_l + \frac{\Omega^2}{\Delta} t'\right)\right)|e_j\rangle$, the corresponding unconventional geometric $2\Theta_l$-phase gates can also be obtained.

In summary, we have specifically studied the approximate implementation of a two-qubit unconventional GQG in the context of a cavity QED system. We have also shown the possibility of having GQGs induced by lth-order squeezed operators. Our proposed gating is insensitive to the atomic spontaneous emission due to large detuning, and in the case of a vacuum cavity mode, the error due to the cavity decay can be avoided.

Acknowledgments

We would like to thank Professor Mang Feng for the enlightening discussion and comments. This work is supported by the Nature Science Foundation of Hunan Province under grant no. 05JJ30005, the National Science Foundation of China under grant nos 10474118 and 1047200, the National Fundamental Research Programme of China under grant no. 2005CB724502, the Science Research Fund of Educational Department of Hunan Province of China under grant no. 05C756 and the Science Research Fund of Hunan Institute of Humanity and Science and Technology under grant no. 2005A001.

References

[1] Zanardi P and Rasetti M 1997 *Phys. Rev. Lett.* **79** 3306
Duan L M and Guo G C 1997 *Phys. Rev. Lett.* **79** 1953
[2] Zanardi P and Rasetti M 1999 *Phys. Lett.* A **264** 94
Paches J, Zanardi P and Rasetti M 2000 *Phys. Rev.* A **61** 01035(R)
[3] Falci G *et al* 2000 *Nature* **407** 355
[4] Duan L-M, Cirac J I and Zoller P 2001 *Science* **292** 1695
Garcia-Ripoll J J and Cirac J I 2003 *Phys. Rev. Lett.* **90** 127902
[5] Jones J A, Vedral V, Ekert A and Castangnoli G 1999 *Nature* **403** 869
[6] Falci G *et al* 2000 *Nature* **407** 355
[7] Wang X B and Keiji M 2001 *Phys. Rev. Lett.* **87** 097901
[8] Solinas P *et al* 2003 *Phys. Rev.* B **67** 121307
Solinas P *et al* 2003 *Phys. Rev.* A **67** 052309
[9] Zhu S L and Wang Z D 2003 *Phys. Rev. Lett.* **91** 187902
Zhu S L, Wang Z D and Zanardi P 2005 *Phys. Rev. Lett.* **94** 100502
[10] Zheng S B 2004 *Phys. Rev.* A **70** 052320
[11] Leibfried D *et al* 2000 *Nature* **422** 412
[12] Luis A 2001 *J. Phys. A: Math. Gen.* **34** 7677
[13] Wang X and Zarnadi P 2002 *Phys. Rev.* A **65** 032327
[14] Jane E, Plenio M B and Joathan D 2002 *Phys. Rev.* A **65** 050302
[15] Rauschenbeutel A *et al* 1999 *Phys. Rev. Lett.* **83** 5166